# Nonlinear Attitude and Position Control of a Micro Quadrotor using Sliding Mode and Backstepping Techniques

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# Topics of this presentation

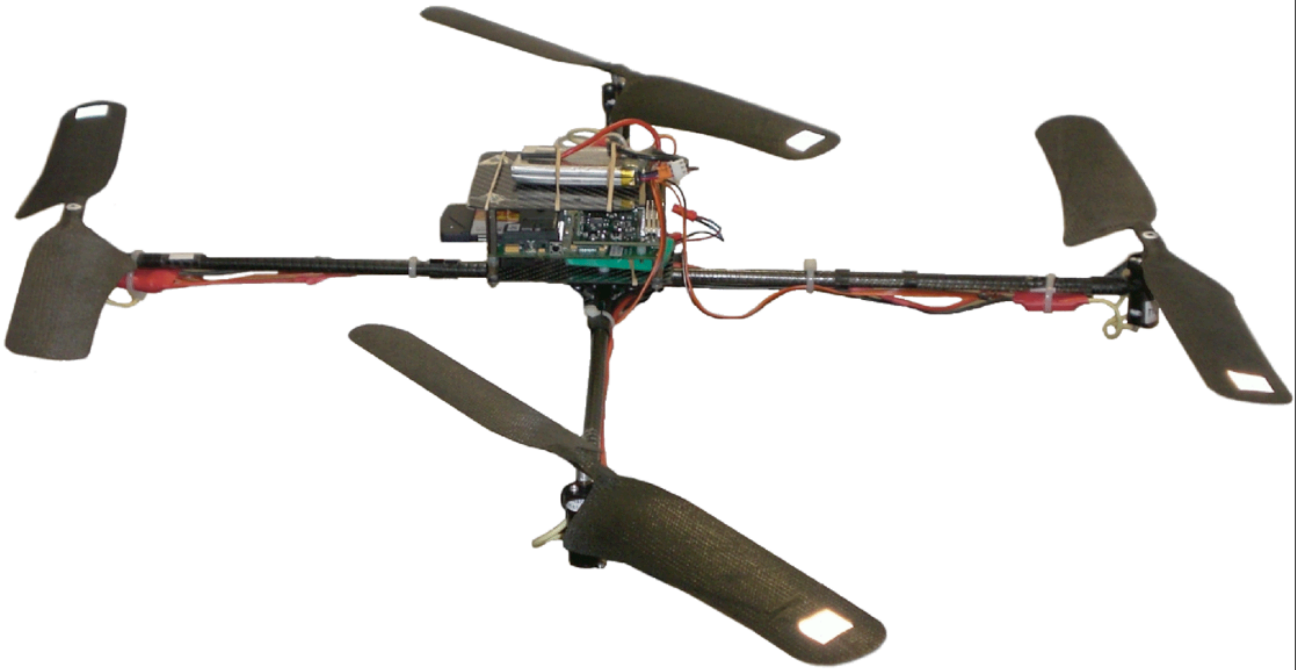
## ***Objectives***

➡ Compare different control strategies for the stabilization of a micro quadri-rotor UAV

➡ Design and implement the most suitable control law

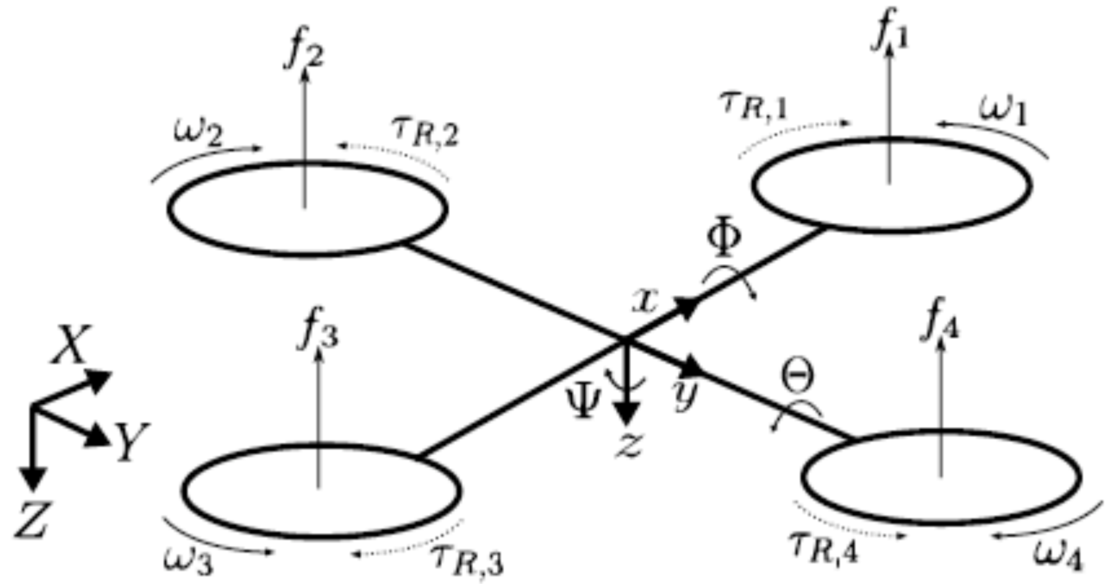
## ***Outline***

1. System modeling
2. Attitude controller
3. Position controller
4. Experimental results
5. Conclusion



*micro Quadri-rotor UAV*

# System modeling




- Configuration vectors

$$\boldsymbol{\xi} = [x, y, z]^{\mathrm{t}}$$

$$\boldsymbol{\eta} = [\Phi, \Theta, \Psi]^{\mathrm{t}}$$

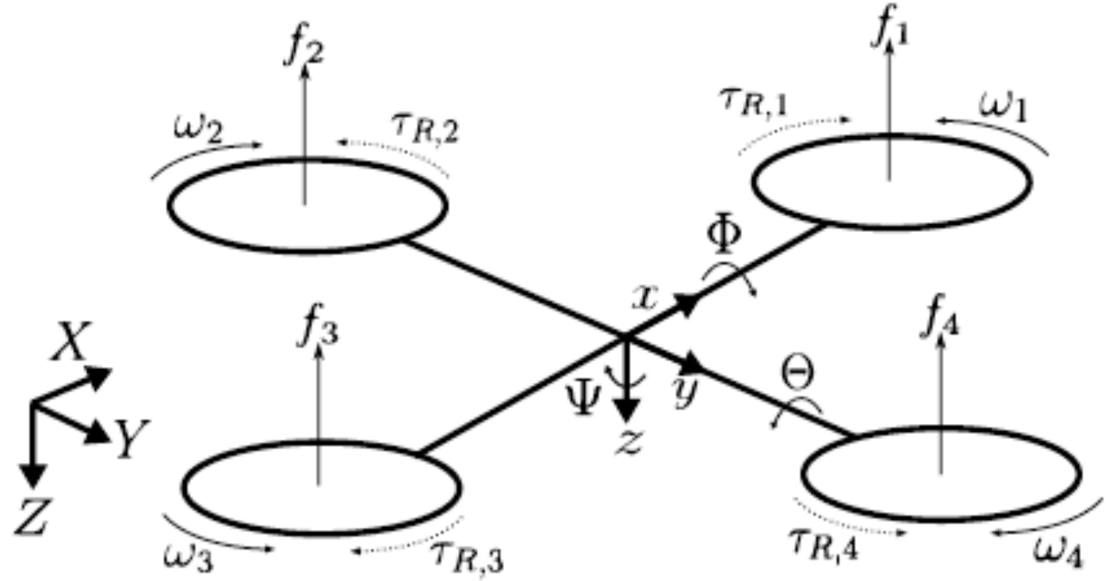
$$\boldsymbol{R} = \begin{pmatrix} C_\Theta C_\Psi & S_\Phi S_\Theta C_\Psi - C_\Phi S_\Psi & C_\Phi S_\Theta C_\Psi + S_\Phi S_\Psi \\ C_\Theta S_\Psi & S_\Phi S_\Theta S_\Psi + C_\Phi C_\Psi & C_\Phi S_\Theta S_\Psi - S_\Phi C_\Psi \\ -S_\Theta & S_\Phi C_\Theta & C_\Phi C_\Theta \end{pmatrix}$$

- Velocity decoupling matrix

$$\boldsymbol{W}(\boldsymbol{\eta}) = \begin{pmatrix} 1 & 0 & -S_\Theta \\ 0 & C_\Phi & S_\Phi C_\Theta \\ 0 & -S_\Phi & C_\Phi C_\Theta \end{pmatrix} \qquad \boldsymbol{\Omega} = \boldsymbol{W}(\boldsymbol{\eta})\,\dot{\boldsymbol{\eta}}$$

# System modeling

- Forces distribution:

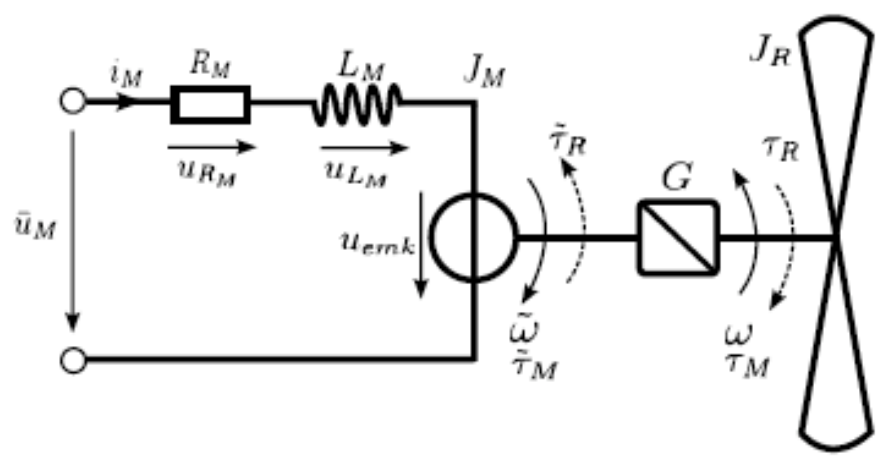



$$\begin{cases} f_i &= k_l\,\omega^2 \\ \tau_{R,i} &= k_d\,\omega^2 \end{cases}$$

$$\begin{bmatrix} \tau_\Phi \\ \tau_\Theta \\ \tau_\Psi \\ F \end{bmatrix} = \begin{pmatrix} 0 & l\,k_l & 0 & -l\,k_l \\ l\,k_l & 0 & -l\,k_l & 0 \\ k_d & -k_d & k_d & -k_d \\ k_l & k_l & k_l & k_l \end{pmatrix} \boldsymbol{\omega}^2 \Rightarrow \mathbf{u} = \begin{bmatrix} \boldsymbol{\tau} \\ \\ F \end{bmatrix} = \begin{bmatrix} \tau_\Phi \\ \tau_\Theta \\ \tau_\Psi \\ F \end{bmatrix}$$

- Actuation model:

$$\dot{\omega} = \frac{k_M}{R_M\,G\,\tilde{J}_{tot}}\,\bar{u}_M - \frac{k_M\,k_{emk}}{R_M\,G\,\tilde{J}_{tot}}\,\omega - \frac{k_d}{G^2\,\eta_G\,\tilde{J}_{tot}}\,\omega^2$$



# System modeling

- Dynamic equations (Euler-Lagrange):

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{\rho}}_i}\right) - \frac{\partial \mathcal{L}}{\partial \boldsymbol{\rho}_i} = Q_i \qquad \text{with} \quad i = 1 \dots 6$$

- Analytical model (with small angles simplification):

$$\left\{\begin{array}{rcl} \ddot{x} & = & -\frac{1}{m_{uav}} F\,(c_\Phi\, c_\Psi\, s_\Theta + s_\Phi\, s_\Psi) \\ \ddot{y} & = & -\frac{1}{m_{uav}} F\,(c_\Phi\, s_\Psi\, s_\Theta - s_\Phi\, c_\Psi) \\ \ddot{z} & = & -\frac{1}{m_{uav}} F\,(c_\Phi\, c_\Theta) + g \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{rcl} \ddot{\Phi} & = & \frac{1}{I_x}\tau_\Phi - \frac{J_R \boldsymbol{\Omega}}{I_x}\dot{\Theta} + \dot{\Theta}\,\dot{\Psi}\left(\frac{I_y - I_z}{I_x}\right) \\ \ddot{\Theta} & = & \frac{1}{I_y}\tau_\Theta + \frac{J_R \boldsymbol{\Omega}}{I_y}\dot{\Phi} + \dot{\Phi}\,\dot{\Psi}\left(\frac{I_z - I_x}{I_y}\right) \\ \ddot{\Psi} & = & \frac{1}{I_z}\tau_\Psi + \dot{\Phi}\,\dot{\Theta}\left(\frac{I_x - I_y}{I_z}\right) \end{array}\right.$$

- Model abstraction:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}$$

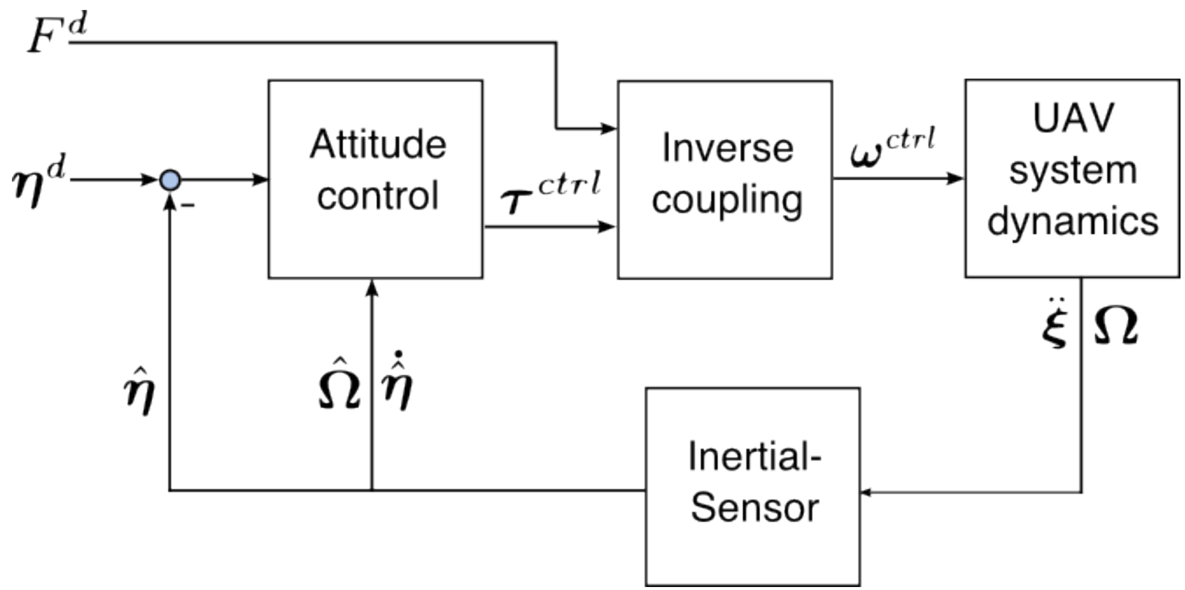
# Control strategies

- Two sub-controllers:
  - Attitude stabilization
  - Position stabilization

- Different control strategies for Attitude Stabilization
  - Quaternion based ***linear feedback*** controller [Tayebi2006]
  - Non-linear ***backstepping*** controller [Bouabdallah2005]
  - Non-linear ***sliding-mode*** controller

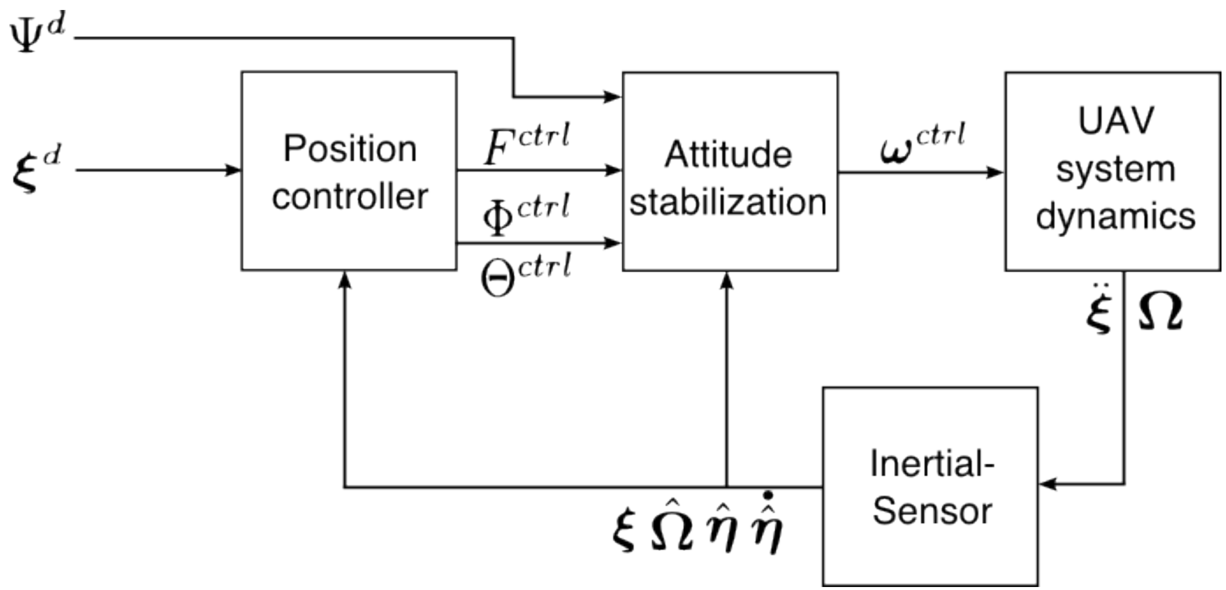
- Backstepping controller for Position Stabilization

# Control strategies



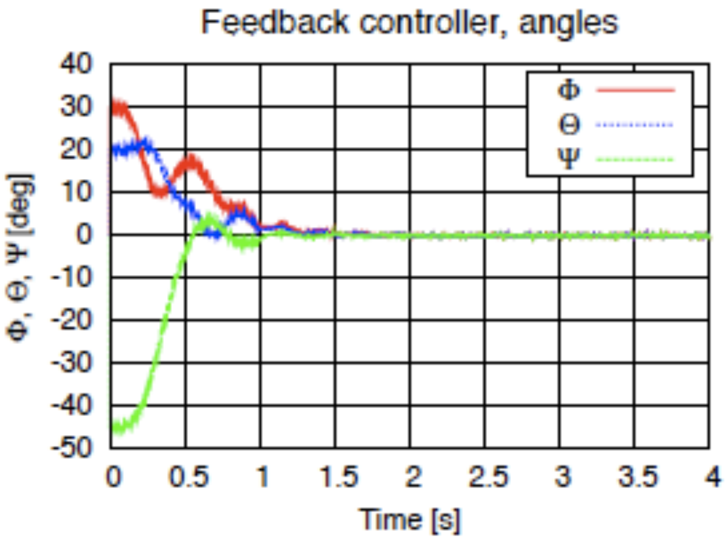

Attitude control architecture

$$\tau^{ctrl} = [\tau_{\Phi}^{ctrl}, \tau_{\Theta}^{ctrl}, \tau_{\Psi}^{ctrl}]^{t}$$




Cascaded control architecture

# Attitude control: Linear feedback controller




- Quaternion based controller
  - Reduced quaternion vector:

$$\boldsymbol{q} = [q_1, q_2, q_3]^{\mathrm{t}}$$

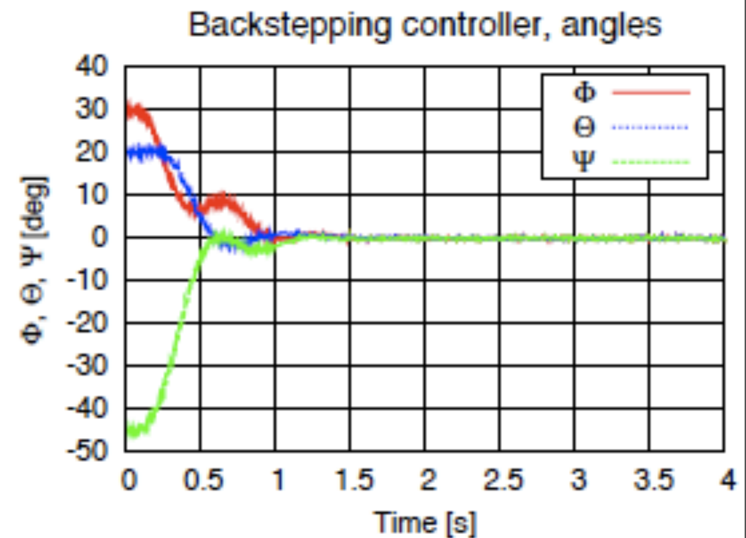
  - Linear feedback law:

$$\boldsymbol{\tau}^{ctrl} = -\mu_p \left(\boldsymbol{q} - \boldsymbol{q}^d\right) - \boldsymbol{\mu}_v \boldsymbol{\Omega}$$

$$\mu_p > 0 \qquad \boldsymbol{\mu}_v = \begin{pmatrix} \mu_{v_1} & 0 & 0 \\ 0 & \mu_{v_2} & 0 \\ 0 & 0 & \mu_{v_3} \end{pmatrix}$$

➡ Controller globally asymptotically stable [Tayebi2005]

# Attitude control: Backstepping controller




***Define non-linear control law based on Lyapunov fonctions***

State vector: $\mathbf{x} = [\Phi,\ \dot{\Phi},\ \Theta,\ \dot{\Theta},\ \Psi,\ \dot{\Psi}]^t$

Transform dynamic state vector: $\mathbf{x} \longrightarrow \mathbf{z}$

$$\begin{cases} z_1 = x_1 - x_1^d \\ z_2 = x_2 - \dot{x}_1^d - \alpha_1(z_1) \end{cases} \qquad \dot{z}_1 = \dot{x}_1 - \dot{x}_1^d = z_2 + \alpha_1(z_1)$$

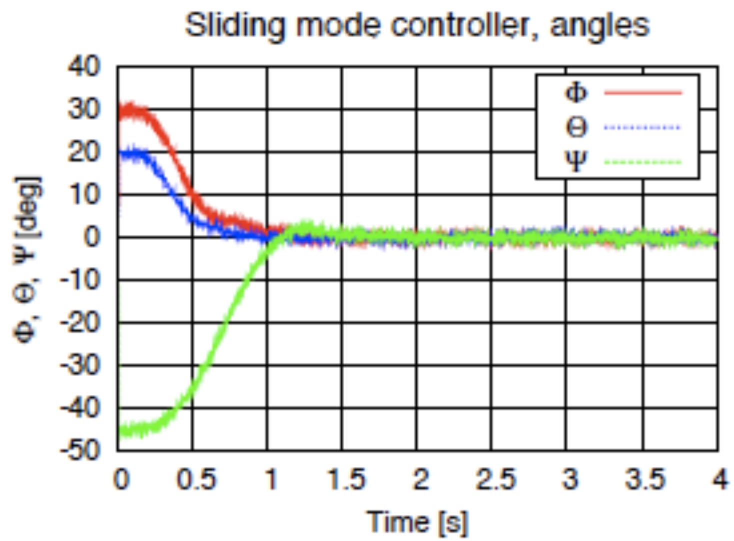
Choose Lyapunov candidate function:

$$V_1 = \frac{1}{2} z_1^2 \quad \text{and} \quad V_2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \qquad \dot{V}_2 < 0 \longrightarrow \alpha_1(z_1)$$

Control law :

$$\boldsymbol{\tau}^{ctrl} = -\boldsymbol{I}\begin{pmatrix} a_1\, a_2 - 1 & 0 & 0 \\ 0 & a_3\, a_4 - 1 & 0 \\ 0 & 0 & a_5\, a_6 - 1 \end{pmatrix}(\boldsymbol{\eta} - \boldsymbol{\eta}^d) - \boldsymbol{I}\begin{pmatrix} a_1 + a_2 & 0 & 0 \\ 0 & a_3 + a_4 & 0 \\ 0 & 0 & a_5 + a_6 \end{pmatrix}\boldsymbol{\Omega}$$

# Attitude control: Sliding mode controller




State vector: $\boldsymbol{x} = [\boldsymbol{\eta}, \dot{\boldsymbol{\eta}}]^{\mathrm{t}}$

System dynamic model: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}$

Switching or sliding manifold:

$$\boldsymbol{S} = \left\{ \boldsymbol{x} \in \mathrm{R}^6 \,|\, \boldsymbol{s}(\boldsymbol{x}) = \mathbf{0} \right\}$$

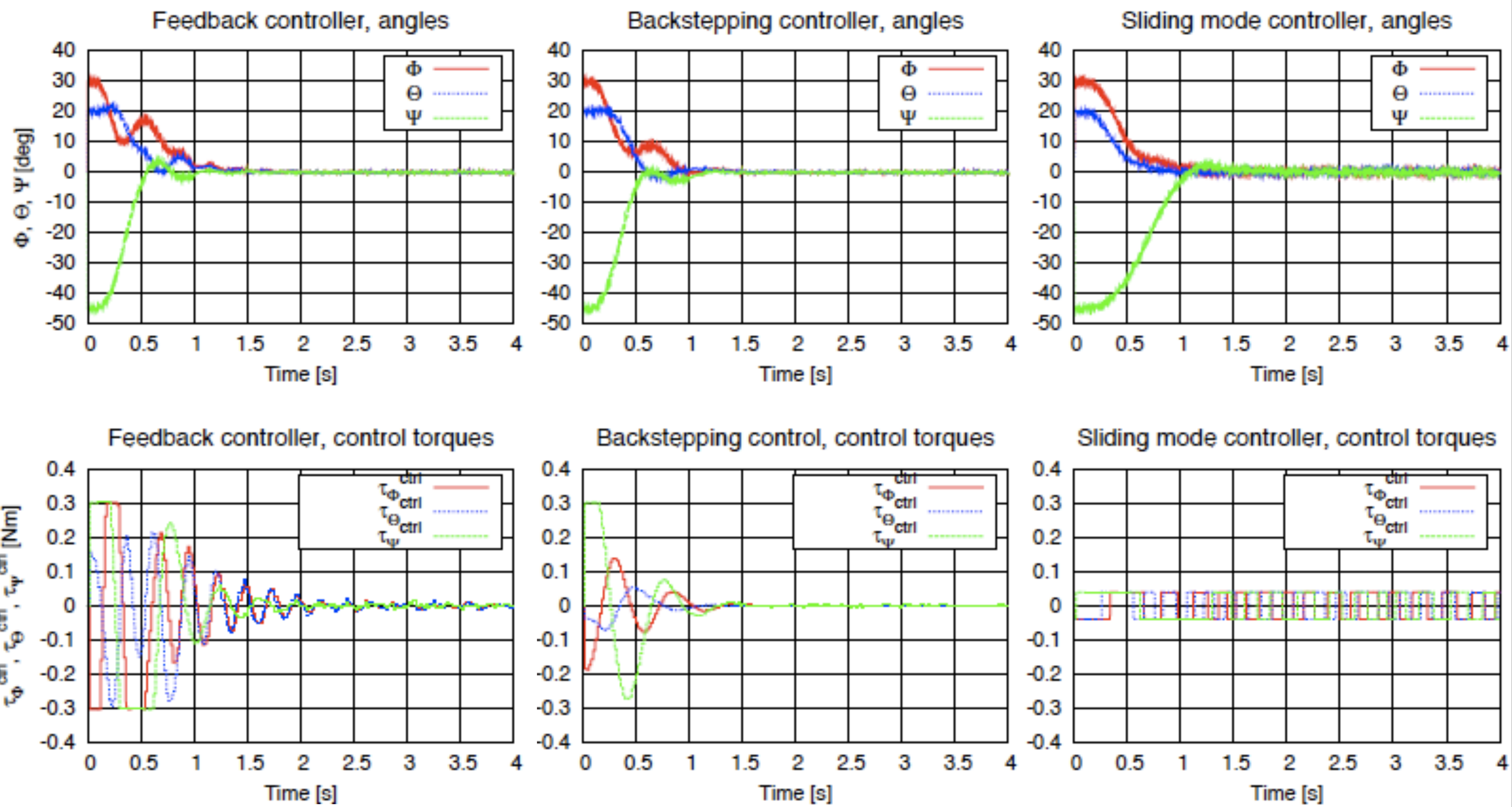
$$\boldsymbol{s}(\boldsymbol{x}) = \boldsymbol{C}_1\, \boldsymbol{e} + \boldsymbol{C}_2\, \dot{\boldsymbol{e}} \qquad \begin{cases} \boldsymbol{e} = \boldsymbol{\eta} - \boldsymbol{\eta}^d \\ \dot{\boldsymbol{e}} = \dot{\boldsymbol{\eta}} - \dot{\boldsymbol{\eta}}^d = \dot{\boldsymbol{\eta}} \end{cases}$$

Sliding-mode control law:

$$\boldsymbol{u}^{ctrl}(\boldsymbol{x}) = -K \,\mathrm{sign}\left(\boldsymbol{s}(\boldsymbol{x})\right) = \begin{cases} \boldsymbol{u}_+(\boldsymbol{x}), \; \boldsymbol{s}(\boldsymbol{x}) > 0 \\ \boldsymbol{u}_-(\boldsymbol{x}), \; \boldsymbol{s}(\boldsymbol{x}) < 0 \end{cases}$$

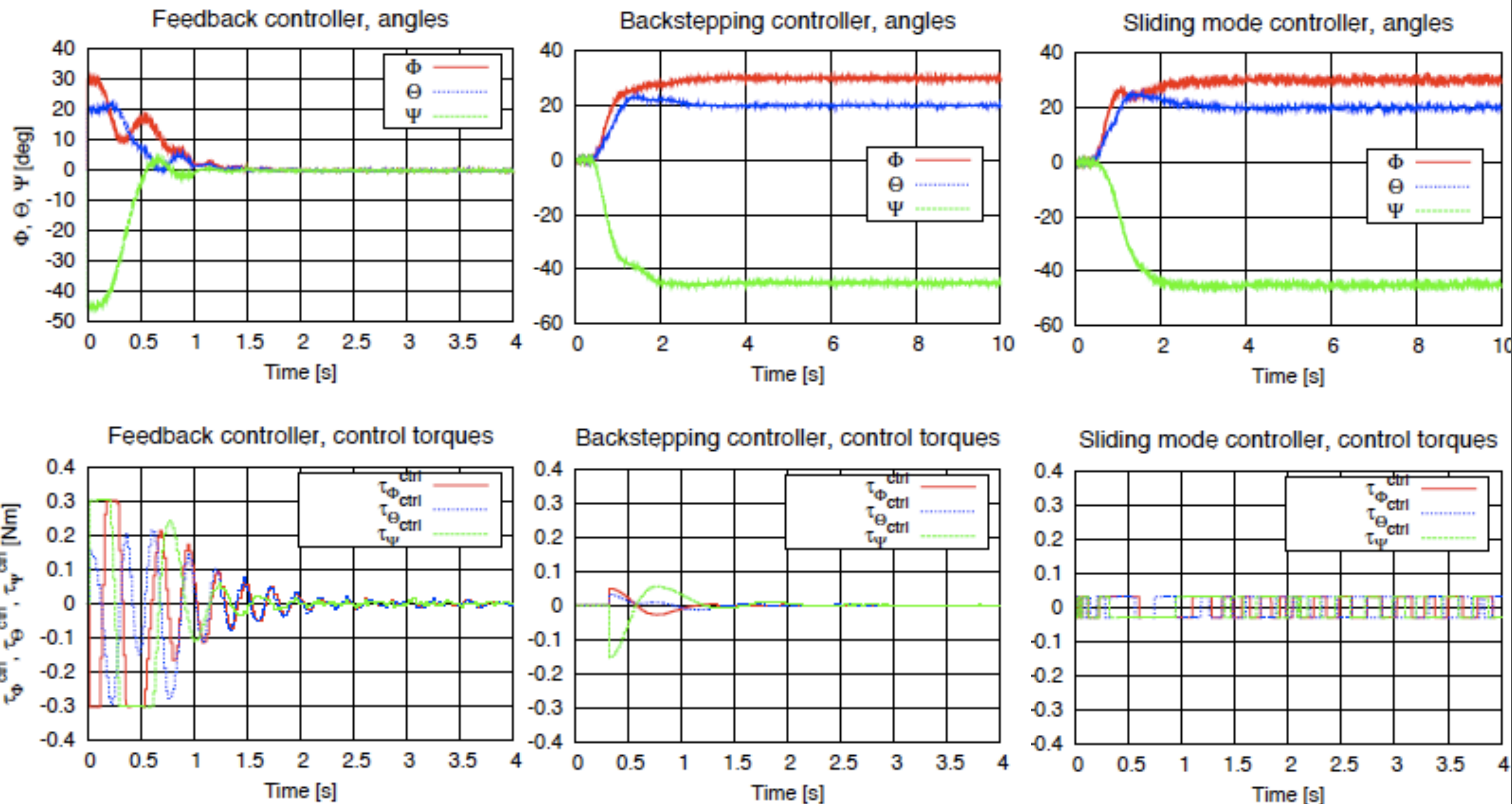
➡ Proof of control law stability (Lyapunov approach): see paper

# Attitude control: analysis and comparison

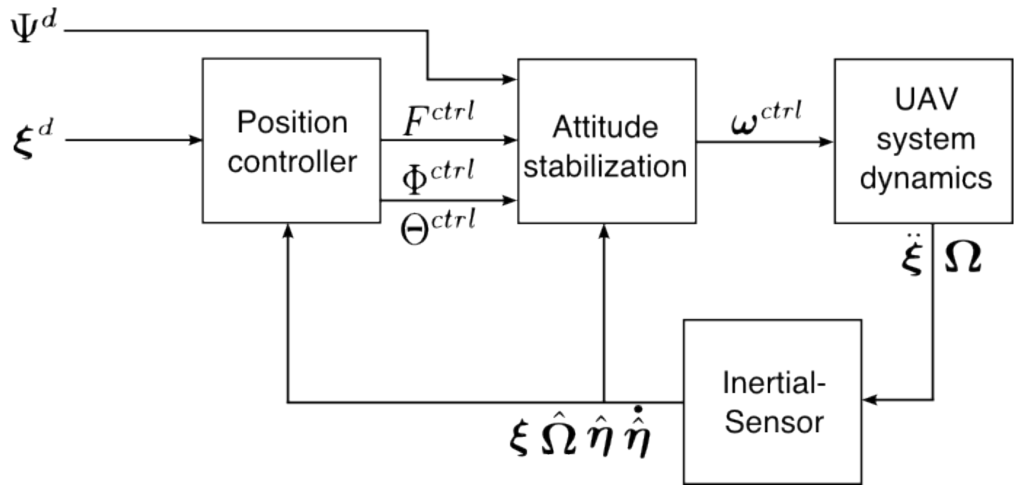


# Attitude control: analysis and comparison

$$\boldsymbol{\eta}^d = [30°, 20°, -45°]^T$$



# Position control: Backstepping controller




Using same approach based on Lyapunov function:

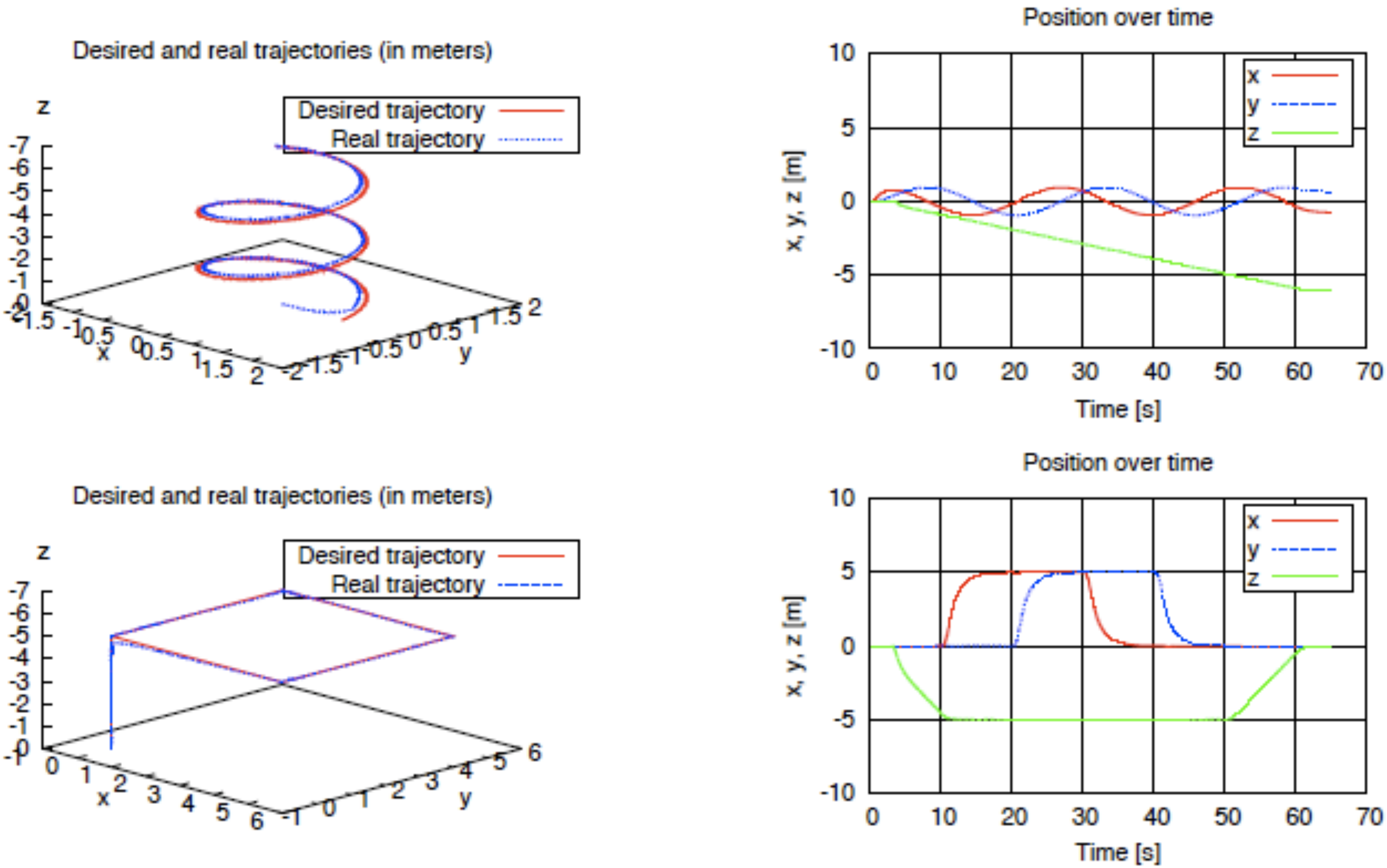
$$\begin{cases} \ddot{x} = r_1 & = & (1 + b_1\, b_2)\,(x - x^d) + (b_1 + b_2)\,\dot{x} \\ \ddot{y} = r_2 & = & (1 + b_3\, b_4)\,(y - y^d) + (b_3 + b_4)\,\dot{y} \\ \ddot{z} = r_3 & = & (1 + b_5\, b_6)\,(z - z^d) + (b_5 + b_6)\,\dot{z} \end{cases}$$

Dynamic model based position controller:

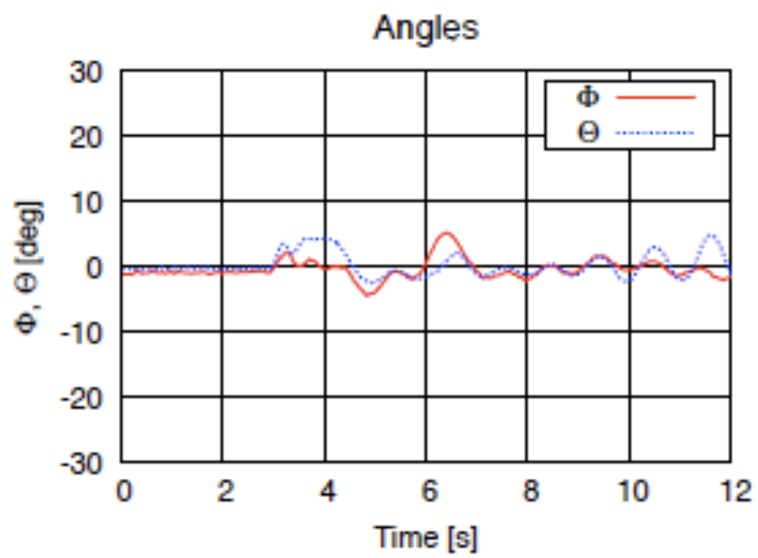
$$\begin{cases} F^{ctrl} & = & \frac{m_{uav}}{c_\Phi\, c_\Theta}\,(r_3 + g) \\ \Phi^{ctrl} & = & \arcsin\left(\frac{m_{uav}}{F^{ctrl}}\,(r_1\, s_{\Psi^d} - r_2\, c_{\Psi^d})\right) \\ \Theta^{ctrl} & = & \arcsin\left(\frac{m_{uav}}{F^{ctrl}\, c_{\Phi^{ctrl}}}\,(r_1\, c_{\Psi^d} + r_2\, s_{\Psi^d})\right) \end{cases}$$

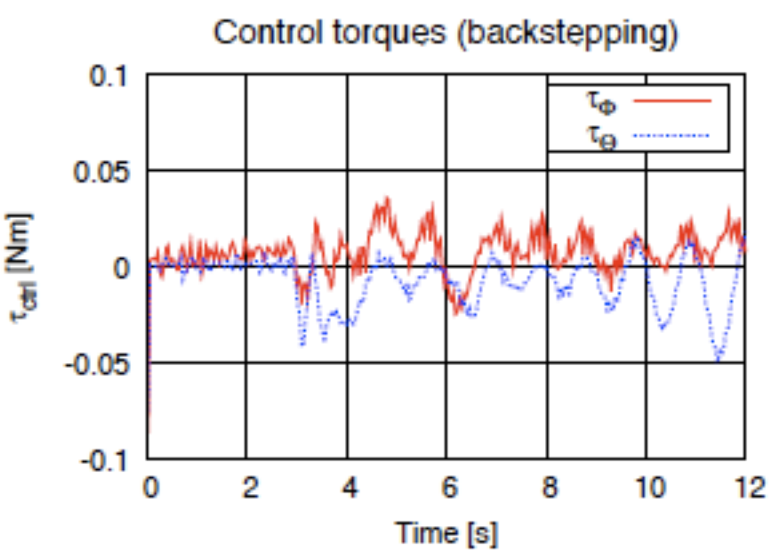
# Position control: Backstepping controller

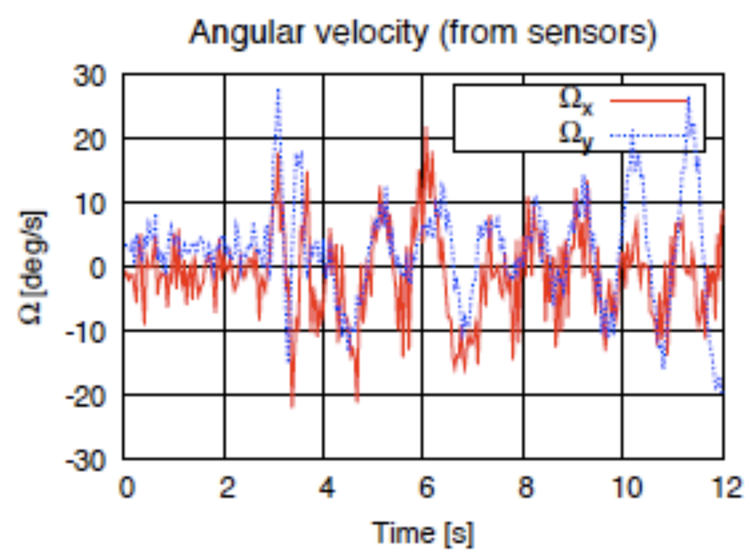
- Simulation results



# First experiments: attitude stabilization



# Conclusions

- Developed a simplified dynamic model for a quadri-rotor
- Analyzed different control strategies for the attitude control
  - Linear feedback law, backstepping, sliding-mode controller
- Proposed a backstepping controller for position stabilization
- Evaluated and tuned the controller using numerical simulations
- First test on a real prototype of quadri-rotor UAV